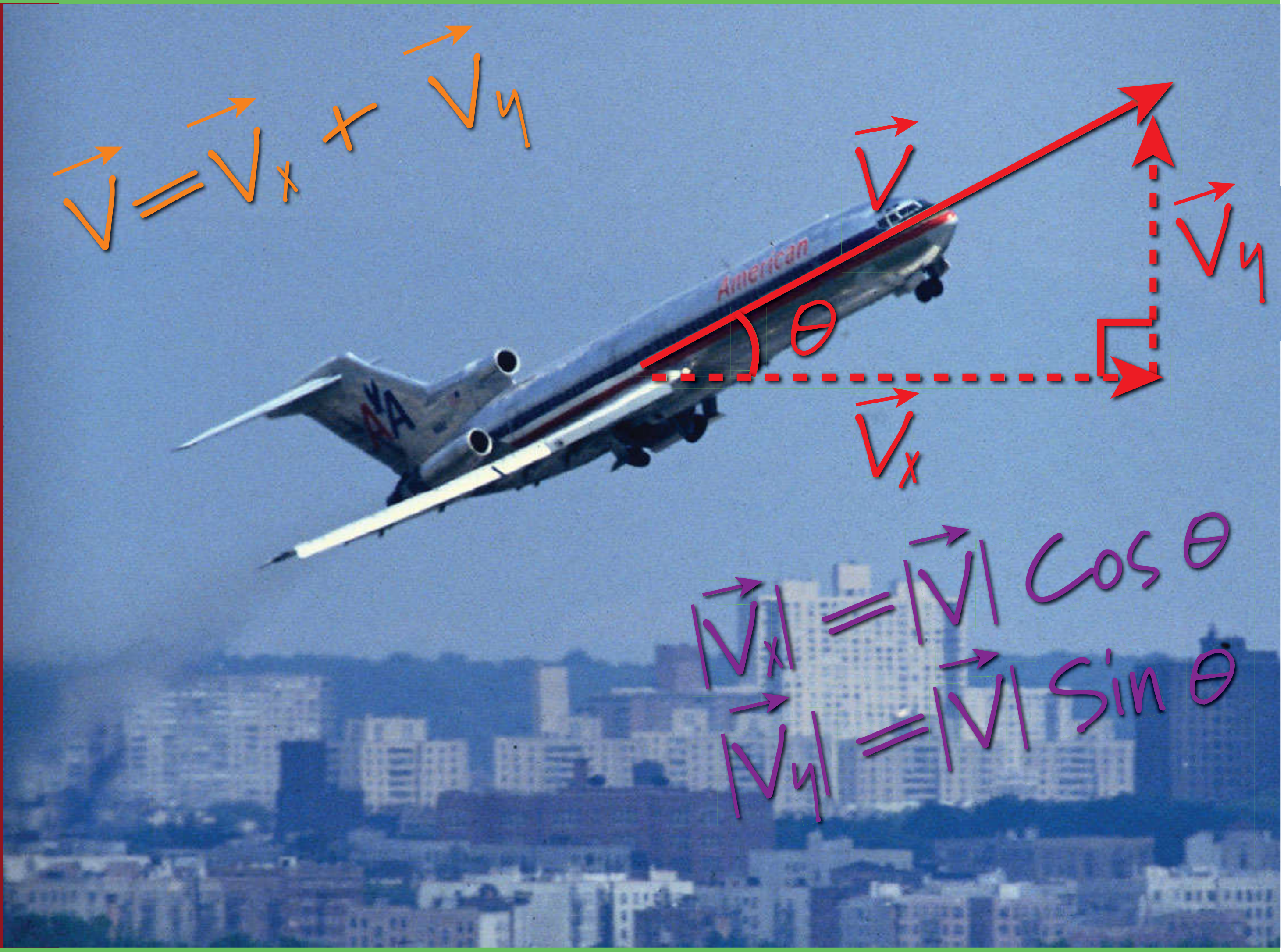
CHAPTER 3

VECTORS



Objectives

The major goals of this chapter are to enable you to:

1. Distinguish between a vector and a scalar quantity.
2. Add vectors graphically.
3. Find the components of a vector.
4. Work with vectors in standard position.
5. Apply the basic concepts of right-triangle trigonometry using displacement vectors.

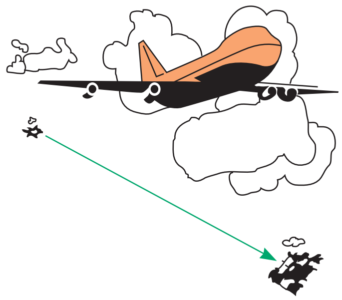
Some physical quantities, called *scalars,* may be described by and involve calculations with numerical quantities alone. Other physical quantities, called *vectors,* require both a numerical quantity and a direction to be completely described and often involve calculations using trigonometry. Vectors are developed in this chapter prior to their use in the following chapters.

3.1 Vectors and Scalars*

Every physical quantity can be classified as either a scalar or a vector quantity. A **scalar** is a quantity that can be completely described by a number (called its magnitude) and a unit. Examples of scalars are length, temperature, and volume. All these quantities can be expressed by a number with the appropriate units. For example, the length of a steel beam is expressed as 18 ft; the temperature at 11:00 A.M. is 15°C; the volume of a room is 300 m^3.

A **vector** is a quantity that requires both *magnitude* (size) and *direction* to be completely described. Examples of vectors are force, displacement, and velocity. To completely describe a force, you must give not only its magnitude (size or amount), but also its direction.

Figure 3.1 Displacement



To describe the change of position of an object, such as an airplane flying from one city to another, we use the term *displacement.* **Displacement** is the net change in position of an object, or the direct distance and direction it moves. For example, to completely describe the flight of a plane between two cities requires both the *distance* between them and the *direction from* the first city *to* the second (Fig. 3.1). *The units of displacement are length units,* such as metres, kilometres, feet, or miles.

Suppose that a friend asks you how to reach your home from school. If you replied that he should walk four blocks, you would not have given him enough information [Fig. 3.2(a)].

Figure 3.2 Displacement involves both a distance and a direction.

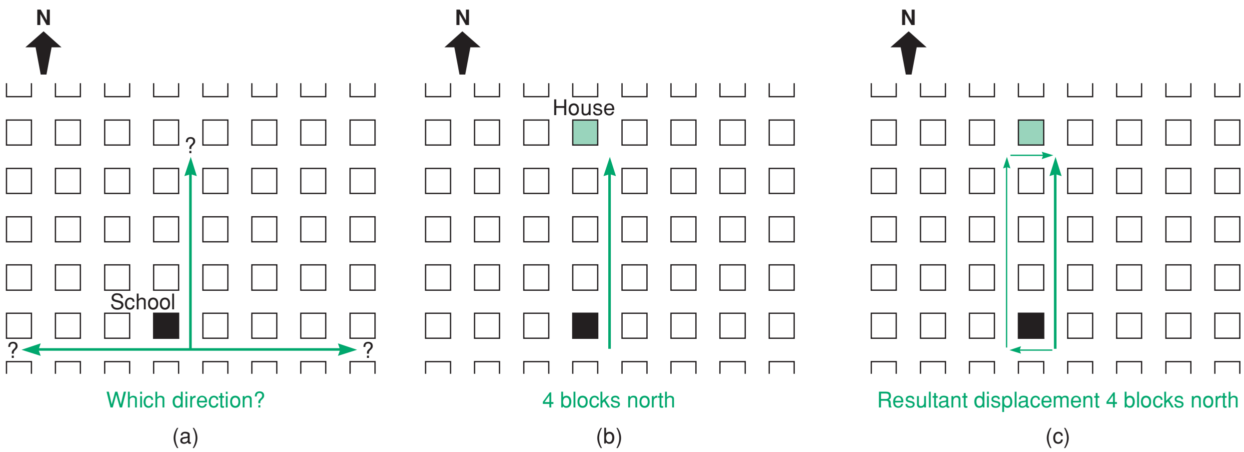
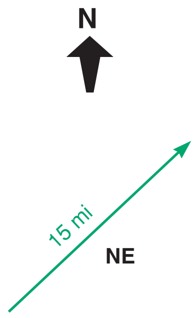


Figure 3.3

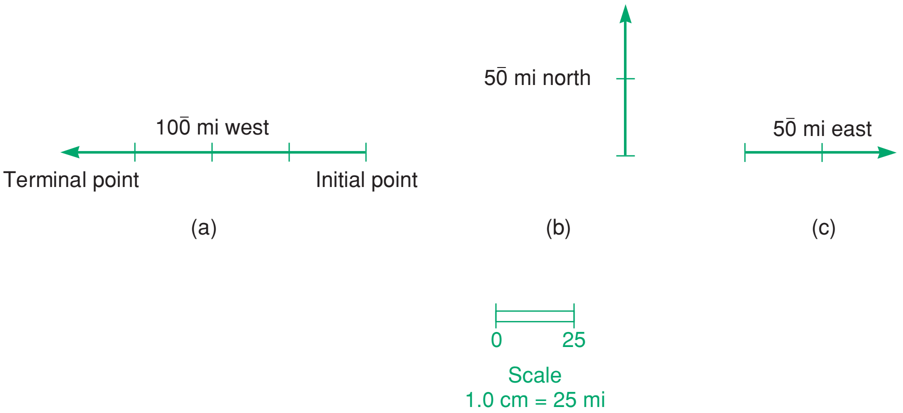


Obviously, you would need to tell him which direction to follow. If you had replied, "Four blocks north," your friend could then find your home [Fig. 3.2(b)]. If your friend decides to walk one block west, four blocks north, and then one block east, he will still arrive at your house. This resultant displacement is the same as if he had walked four blocks north [Fig. 3.2(c)].

The magnitude of the displacement vector "15 miles NE" is 15 miles and its direction is northeast (Fig. 3.3).

*Right-triangle trigonometry is developed in Appendix A.5, and instructions for using sin, cos, and tan keys on a scientific calculator are included in Appendix B.3 for those who have not studied this before or who need a review.

Figure 3.4 Use a scale to draw the proper length of a given vector.



To represent a vector in a diagram, we draw an arrow that points in the correct direction. The magnitude of the vector is indicated by the length of the arrow. We usually choose a scale, such as 1.0 cm = 25 mi, for this purpose (Fig. 3.4). Thus, a displacement of $10\bar{0}$ mi west is drawn as an arrow (pointing west) 4.0 cm long [Fig. 3.4(a)] since

$$10\bar{0}\ \cancel{\text{mi}} \times \frac{1.0\ \text{cm}}{25\ \cancel{\text{mi}}} = 4.0\ \text{cm}$$

Displacements of $5\bar{0}$ mi north [Fig. 3.4(b)] and $5\bar{0}$ mi east [Fig. 3.4(c)] using the same scale are also shown.

EXAMPLE 1

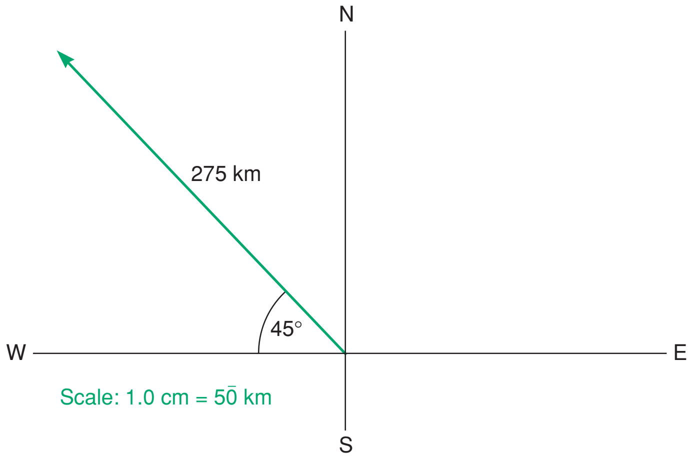
Using the scale 1.0 cm = $5\bar{0}$ km, draw the displacement vector 275 km at 45° north of west.

First, find the length of the vector.

$$275\ \cancel{\text{km}} \times \frac{1.0\ \text{cm}}{5\bar{0}\ \cancel{\text{km}}} = 5.5\ \text{cm}$$

Then draw the vector at an angle 45° north of west (Fig. 3.5).

Figure 3.5



EXAMPLE 2

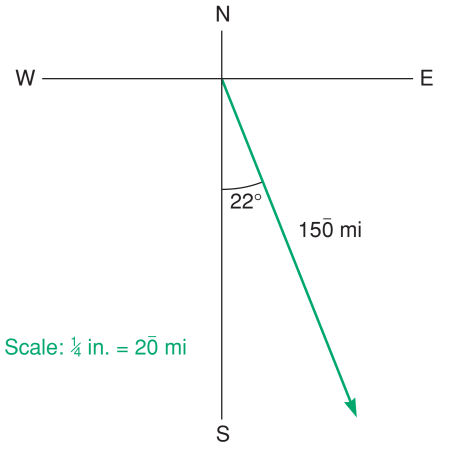
Using the scale $\frac{1}{4}$ in. $= 2\bar{0}$ mi, draw the displacement vector $15\bar{0}$ mi at 22° east of south.

First, find the length of the vector.

$$15\bar{0}\ \cancel{\text{mi}} \times \frac{\frac{1}{4}\ \text{in.}}{2\bar{0}\ \cancel{\text{mi}}} = 1\tfrac{7}{8}\ \text{in.}$$

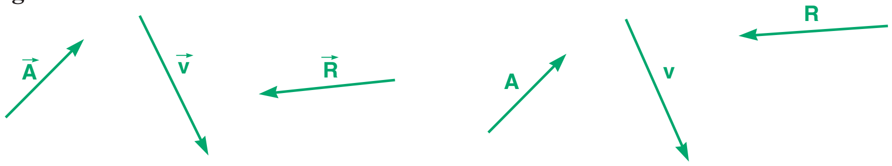
Then draw the vector at 22° east of south (Fig. 3.6).

Figure 3.6



A vector may be denoted by a single letter with a small arrow above, such as $\vec{A}$, $\vec{v}$, or $\vec{R}$ [Fig. 3.7(a)]. This notation is especially useful when writing vectors on paper or on a chalkboard. In this book we use the traditional boldface type to denote vectors, such as **A**, **v**, or **R** [Fig. 3.7(b)]. The length of vector $\vec{A}$ is written $|\vec{A}|$; the length of vector **A** is written |**A**|.

Figure 3.7



(a) Vector quantities $\vec{A}$, $\vec{v}$, and $\vec{R}$ usually have arrows when they are written on paper or on a chalkboard.

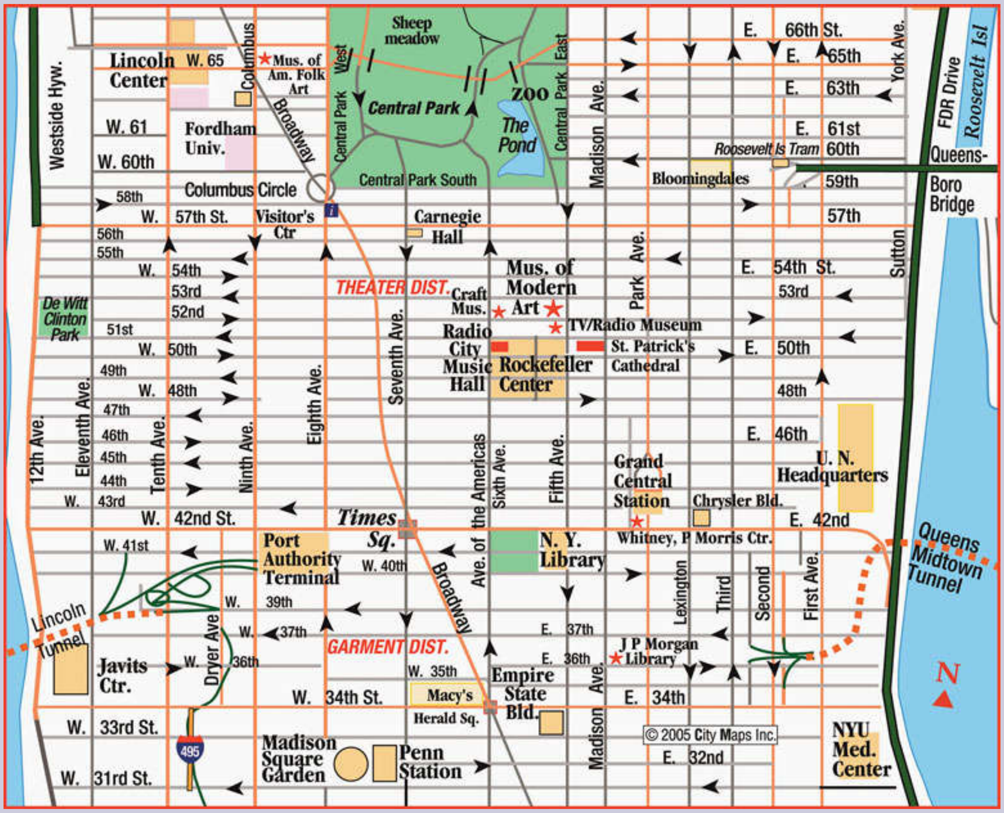
(b) Vector quantities **A**, **v**, and **R** usually are written in boldface type in textbooks.

TRY THIS ACTIVITY

New York Vectors

In 1811, a comprehensive plan was mapped out to create a rectangular grid of roadways on the island of Manhattan in New York City. As a result, giving directions in Manhattan can be done in a number of different ways while still achieving the same result. Assuming that the streets and avenues in the map in Fig. 3.8 are at right angles with one another and that the distance between streets is 0.05 mi and the distance between avenues in 0.20 mi, determine three different ways that someone could travel from Macy's at Herald Square to Times Square. What would be the distance traveled and the displacement for each of these paths?

Figure 3.8 A midtown Manhattan map is a good way to demonstrate the usefulness of vectors.



PROBLEMS 3.1

Using the scale 1.0 cm = $5\bar{0}$ km, find the length of the vector that represents each displacement.

1. Displacement $10\bar{0}$ km east — length = _____ cm
2. Displacement 125 km south — length = _____ cm
3. Displacement $14\bar{0}$ km at 45° east of south — length = _____ cm
4. Displacement $26\bar{0}$ km at 30° south of west — length = _____ cm
5. Displacement 315 km at 65° north of east — length = _____ cm
6. Displacement 187 km at 17° north of west — length = _____ cm

7–12. Draw the vectors in Problems 1 through 6 using the scale indicated.

Using the scale $\frac{1}{4}$ in. = $2\bar{0}$ mi, find the length of the vector that represents each displacement.

13. Displacement $10\bar{0}$ mi west — length = _____ in.
14. Displacement $17\bar{0}$ mi north — length = _____ in.
15. Displacement $21\bar{0}$ mi at 45° south of west — length = _____ in.
16. Displacement 145 mi at $6\bar{0}$° north of east — length = _____ in.
17. Displacement 75 mi at 25° west of north — length = _____ in.
18. Displacement $16\bar{0}$ mi at 72° west of south — length = _____ in.

19–24. Draw the vectors in Problems 13 through 18 using the scale indicated.

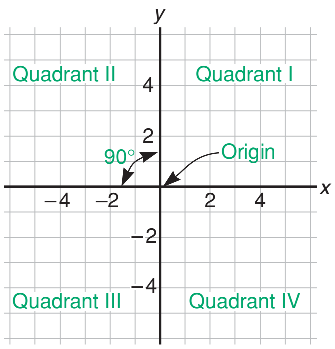
3.2 Components of a Vector

Before we study vectors further, we need to discuss components of vectors and the number plane. The **number plane** (sometimes called the *Cartesian coordinate system,* after René Descartes) consists of a horizontal line called the *x*-axis and a vertical line called the *y*-axis intersecting at a right angle at a point called the *origin* as shown in Fig. 3.9. These two lines divide

René Descartes (1596–1650),

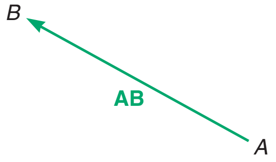
mathematician and philosopher, was born in France. He founded analytic or coordinate geometry, often called Cartesian geometry, and made major contributions in optics.

Figure 3.9 Number plane



the number plane into four quadrants, which we label as quadrants I, II, III, and IV. The x-axis contains positive numbers to the right of the origin and negative numbers to the left of the origin. The y-axis contains positive numbers above the origin and negative numbers below the origin.

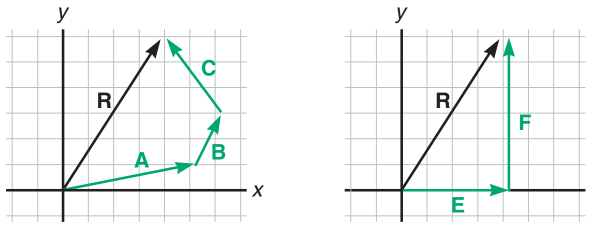
Figure 3.10 Vector from A to B



Graphically, a vector is represented by a directed line segment. The length of the line segment indicates the magnitude of the quantity. An arrowhead indicates the direction. If A and B are the end points of a line segment as in Fig. 3.10, the symbol **AB** denotes the *vector from A to B*. Point A is called the *initial point*. Point B is called the *terminal point* or *end point* of the vector. Vector **BA** has the same length as vector **AB** but has the opposite direction. Vectors may also be denoted by a single letter, such as **u**, **v**, or **R**.

The sum of two or more vectors is called the **resultant vector**. When two or more vectors are added, each of these vectors is called a **component** of the resultant vector. The components of vector **R** in Fig. 3.11(a) are vectors **A**, **B**, and **C**. ***Note:*** A vector may have more than one set of component vectors. The components of vector **R** in Fig. 3.11(b) are vectors **E** and **F**.

Figure 3.11

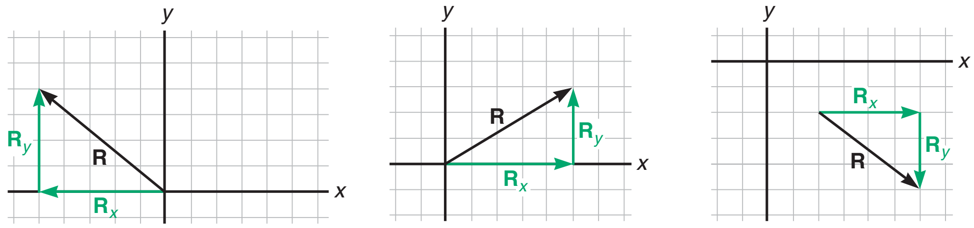


(a) Vectors **A**, **B**, and **C** are components of the resultant vector **R**.

(b) Vector **E** is a horizontal component and vector **F** is a vertical component of the resultant vector **R**.

We are often interested in the components of a vector that are perpendicular to each other and that are on or parallel to the x- and y-axes. In particular, we are interested in the type of component vectors shown in Fig. 3.11(b) (component vectors **E** and **F**). The horizontal component vector that lies on or is parallel to the x-axis is called the ***x*-component**. The vertical component vector that lies on or is parallel to the y-axis is called the ***y*-component**. Three examples are shown in Fig. 3.12.

Figure 3.12



(a) (b) (c)

$\mathbf{R}_x$ = the x-component of vector **R**
$\mathbf{R}_y$ = the y-component of vector **R**

The x- and y-components of vectors can also be expressed as signed numbers. The absolute value of the signed number corresponds to the magnitude (length) of the component vector. The sign of the number corresponds to the direction of the component as follows:

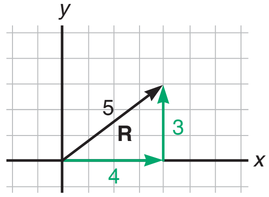
x-component	*y*-component
+, if right	+, if up
−, if left	−, if down

EXAMPLE 1

Find the x- and y-components of vector **R** in Fig. 3.13.

$$\mathbf{R}_x = x\text{-component of } \mathbf{R} = +4$$
$$\mathbf{R}_y = y\text{-component of } \mathbf{R} = +3$$

Figure 3.13



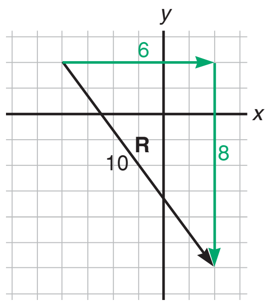
EXAMPLE 2

Find the x- and y-components of vector **R** in Fig. 3.14.

$$\mathbf{R}_x = x\text{-component of } \mathbf{R} = +6$$
$$\mathbf{R}_y = y\text{-component of } \mathbf{R} = -8$$

(The y-component points in a negative direction.)

Figure 3.14



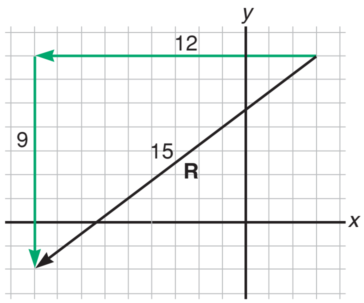
EXAMPLE 3

Find the x- and y-components of vector **R** in Fig. 3.15.

$$\mathbf{R}_x = -12$$
$$\mathbf{R}_y = -9$$

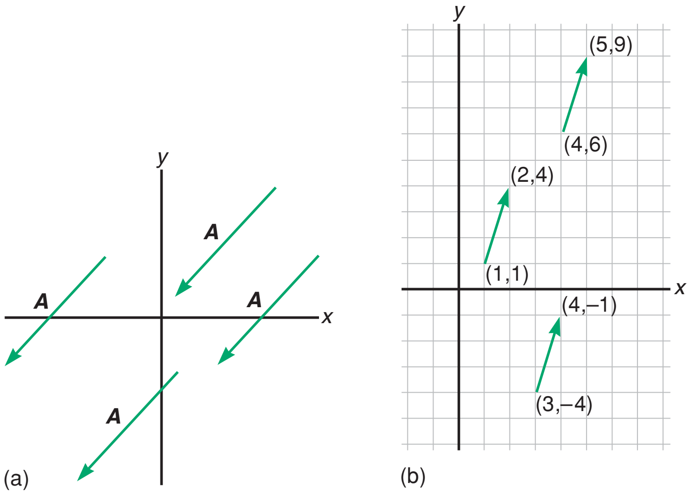
(Both x- and y-components point in a negative direction.)

Figure 3.15



A vector may be placed in any position in the number plane as long as its magnitude and direction are not changed. The vectors in each set in Fig. 3.16 are equal because they have the same magnitude (length) and the same direction.

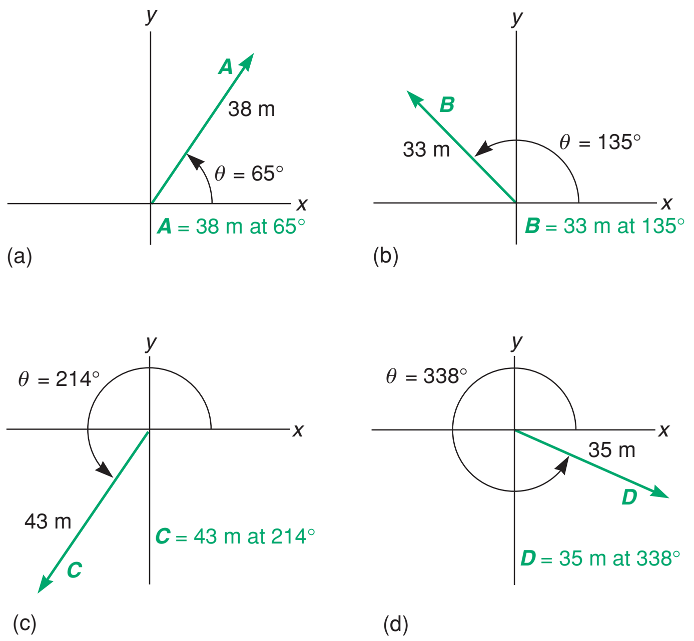
Figure 3.16



A vector is in **standard position** when its initial point is at the origin of the number plane. A vector in standard position is expressed in terms of its magnitude (length) and its

angle θ, where θ *is measured counterclockwise from the positive x-axis to the vector.* The vectors shown in Fig. 3.17 are in standard position.

Figure 3.17 Vectors in standard position



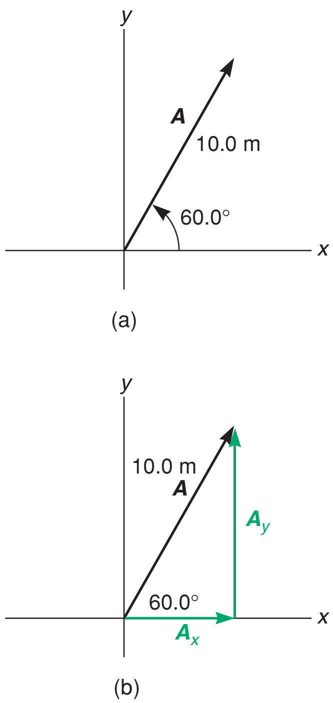
EXAMPLE 4

Finding the Components of a Vector

Find the x- and y-components of the vector $\mathbf{A} = 10.0$ m at $60.0°$.

First, draw the vector in standard position [Fig. 3.18(a)]. Then, draw a right triangle where the legs represent the x- and y-components [Fig. 3.18(b)]. The absolute value of the x-component of the vector is the length of the side adjacent to the $60.0°$ angle. Therefore, to find the x-component,

Figure 3.18



$$\cos 60.0° = \frac{\text{side adjacent to } 60.0°}{\text{hypotenuse}} = \frac{|\mathbf{A}_x|}{10.0 \text{ m}}$$

$$\cos 60.0° = \frac{|\mathbf{A}_x|}{10.0 \text{ m}}$$

$$(\cos 60.0°)(10.0 \text{ m}) = \left(\frac{|\mathbf{A}_x|}{10.0 \text{ m}}\right)(10.0 \text{ m}) \qquad \text{Multiply both sides by 10.0 m.}$$

$$5.00 \text{ m} = |\mathbf{A}_x|$$

Since the x-component is pointing in the positive x-direction, $\mathbf{A}_x = +5.00$ m.

The absolute value of the y-component of the vector is the length of the side opposite the $60.0°$ angle. Therefore, to find the y-component,

$$\sin 60.0° = \frac{\text{side opposite } 60.0°}{\text{hypotenuse}} = \frac{|\mathbf{A}_y|}{10.0 \text{ m}}$$

$$\sin 60.0° = \frac{|\mathbf{A}_y|}{10.0 \text{ m}}$$

$$(\sin 60.0°)(10.0 \text{ m}) = \left(\frac{|\mathbf{A}_y|}{10.0 \text{ m}}\right)(10.0 \text{ m}) \qquad \text{Multiply both sides by 10.0 m.}$$

$$8.66 \text{ m} = |\mathbf{A}_y|$$

Since the y-component is pointing in the positive y-direction, $\mathbf{A}_y = +8.66$ m.

EXAMPLE 5

Find the x- and y-components of the vector $\mathbf{B} = 13.0$ km at $220.0°$.

First, draw the vector in standard position [Fig. 3.19(a)]. Then, complete a right triangle with the x- and y-components being the two legs [Fig. 3.19(b)].

We will let angle α (Greek letter alpha) be the acute angle (an angle whose measure is less than 90°) between the vector in standard position and the x-axis.

Find angle α as follows:

$$180° + \alpha = 220.0°$$
$$\alpha = 40.0°$$

The absolute value of the x-component is the length of the side adjacent to angle α. Therefore, to find the x-component,

$$\cos\alpha = \frac{\text{side adjacent to } \alpha}{\text{hypotenuse}}$$

$$\cos 40.0° = \frac{|\mathbf{B}_x|}{13.0 \text{ km}}$$

$$(\cos 40.0°)(13.0 \text{ km}) = \left(\frac{|\mathbf{B}_x|}{13.0 \text{ km}}\right)(13.0 \text{ km}) \qquad \text{Multiply both sides by 13.0 km.}$$

$$9.96 \text{ km} = |\mathbf{B}_x|$$

Since the x-component is pointing in the negative x-direction, $\mathbf{B}_x = -9.96$ km.

The absolute value of the y-component of the vector is the length of the side opposite angle α. Therefore, to find the y-component,

$$\sin\alpha = \frac{\text{side opposite } \alpha}{\text{hypotenuse}}$$

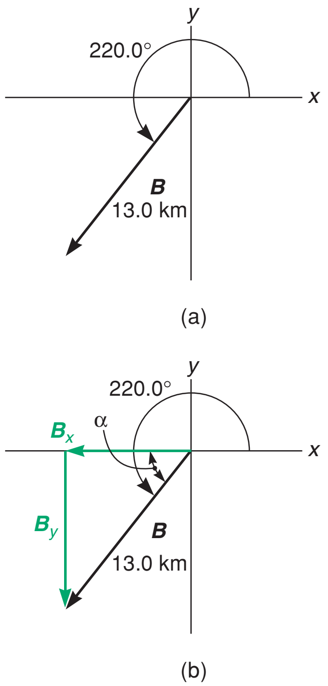
$$\sin 40.0° = \frac{|\mathbf{B}_y|}{13.0 \text{ km}}$$

$$(\sin 40.0°)(13.0 \text{ km}) = \left(\frac{|\mathbf{B}_y|}{13.0 \text{ km}}\right)(13.0 \text{ km}) \qquad \text{Multiply both sides by 13.0 km.}$$

$$8.36 \text{ km} = |\mathbf{B}_y|$$

Since the y-component is pointing in the negative y-direction, $\mathbf{B}_y = -8.36$ km.

Figure 3.19



In general, find the x- and y-components of a vector as follows. First, draw any vector $\mathbf{A}$ in standard position; then, draw its x- and y-components as shown in Fig. 3.20. Use the right triangle to find the x-component as follows:

$$\cos\alpha = \frac{\text{side adjacent to } \alpha}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{|\mathbf{A}_x|}{|\mathbf{A}|}$$

$$|\mathbf{A}|(\cos\alpha) = \left(\frac{|\mathbf{A}_x|}{|\mathbf{A}|}\right)|\mathbf{A}| \qquad \text{Multiply both sides by } |\mathbf{A}|.$$

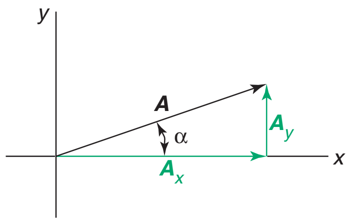
$$|\mathbf{A}|(\cos\alpha) = |\mathbf{A}_x|$$

Similarly, we use the right triangle to find the y-component as follows:

$$\sin\alpha = \frac{\text{side opposite } \alpha}{\text{hypotenuse}}$$

$$\sin\alpha = \frac{|\mathbf{A}_y|}{|\mathbf{A}|}$$

Figure 3.20



Vector $\mathbf{v}$ in standard position with its horizontal component $\mathbf{v}_x$ and its vertical component $\mathbf{v}_y$

$$|\mathbf{A}|(\sin \alpha) = \left(\frac{|\mathbf{A}_y|}{|\mathbf{A}|}\right)|\mathbf{A}| \qquad \text{Multiply both sides by } |\mathbf{A}|.$$

$$|\mathbf{A}|(\sin \alpha) = |\mathbf{A}_y|$$

The signs of the x- and y-components are determined by the quadrants in which the vector in standard position lies.

In general:

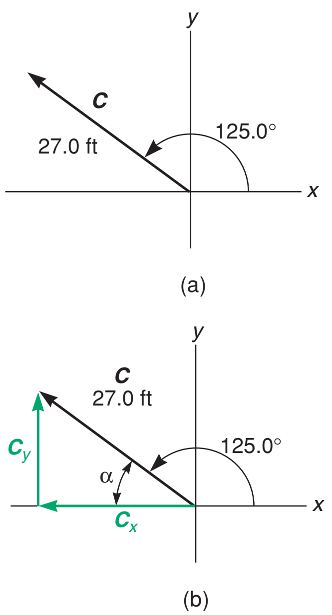
> To find the x- and y-components of a vector **A** given in standard position:
>
> **1.** Complete the right triangle with the legs being the x- and y-components of the vector.
>
> **2.** Find the lengths of the legs of the right triangle as follows:
>
> $$|\mathbf{A}_x| = |\mathbf{A}|(\cos \alpha)$$
> $$|\mathbf{A}_y| = |\mathbf{A}|(\sin \alpha)$$
>
> where angle α is the acute angle between vector **A** in standard position and the x-axis.
>
> **3.** Determine the signs of the x- and y-components.

EXAMPLE 6

Find the x- and y-components of the vector $\mathbf{C} = 27.0$ ft at 125.0°.

First, draw the vector in standard position [Fig. 3.21(a)]. Then, complete a right triangle with the x- and y-components being the two legs [Fig. 3.21(b)]. Find angle α as follows:

Figure 3.21



$$\alpha + 125.0° = 180°$$
$$\alpha = 55.0°$$

Next, find the x-component as follows:

$$|\mathbf{C}_x| = |\mathbf{C}|(\cos \alpha)$$
$$|\mathbf{C}_x| = (27.0 \text{ ft})(\cos 55.0°)$$
$$= 15.5 \text{ ft}$$

Since the x-component is pointing in the negative x-direction,

$$\mathbf{C}_x = -15.5 \text{ ft}$$

Then, find the y-component as follows:

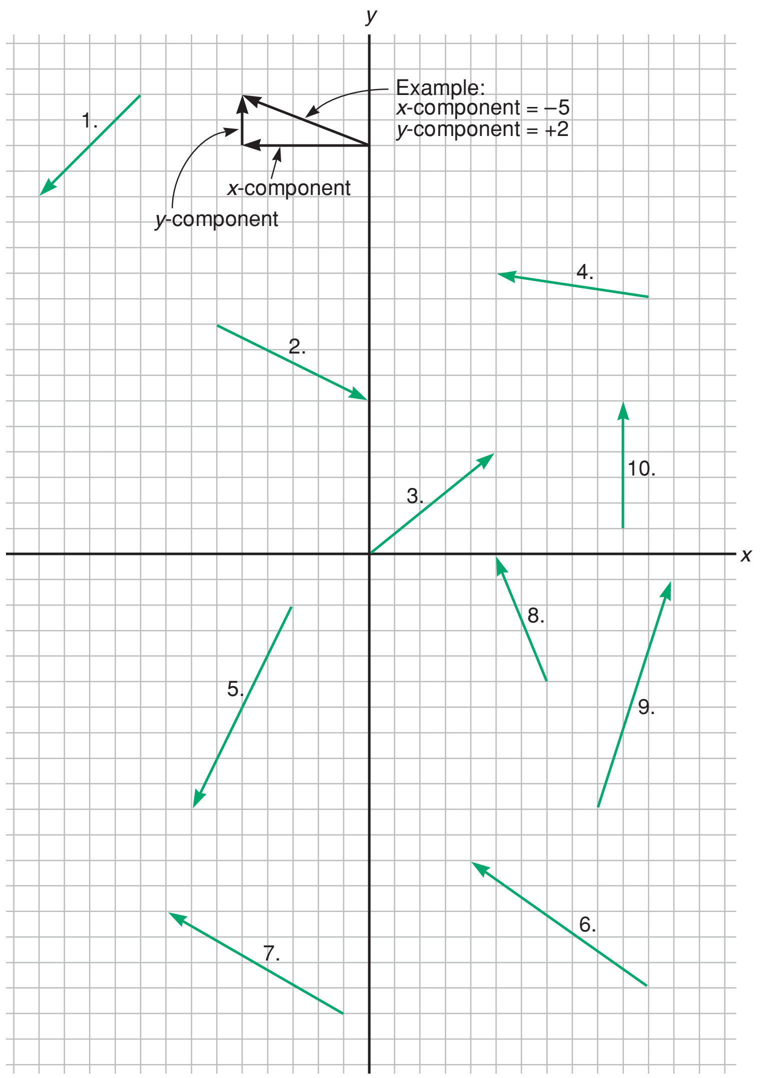
$$|\mathbf{C}_y| = |\mathbf{C}|(\sin \alpha)$$
$$|\mathbf{C}_y| = (27.0 \text{ ft})(\sin 55.0°)$$
$$= 22.1 \text{ ft}$$

Since the y-component is pointing in the positive y-direction,

$$\mathbf{C}_y = +22.1 \text{ ft}$$

PROBLEMS 3.2

Find the x- and y-components of each vector in the following diagram. (Express them as signed numbers and then graph them as vectors.)

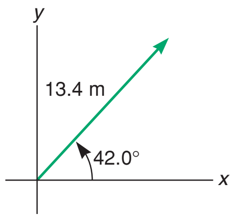


Make a sketch of each vector in standard position. Use the scale 1.0 cm = $1\bar{0}$ m.

11. **A** = $2\bar{0}$ m at 25°
12. **B** = 25 m at 125°
13. **C** = 25 m at 245°
14. **D** = $2\bar{0}$ m at 345°
15. **E** = 15 m at 105°
16. **F** = 35 m at 291°
17. **G** = $3\bar{0}$ m at 405°
18. **H** = 25 m at 525°

Find the x- and y-components of each vector.

19.



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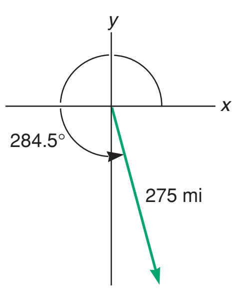

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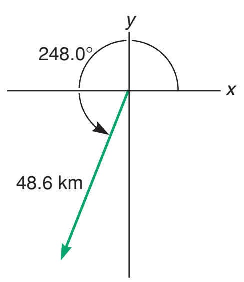

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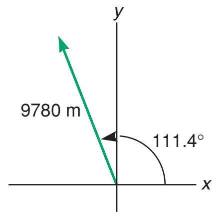
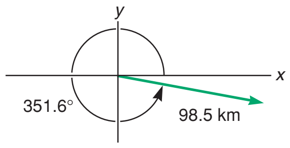
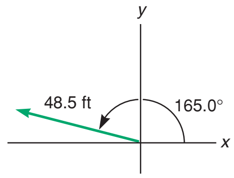


23.



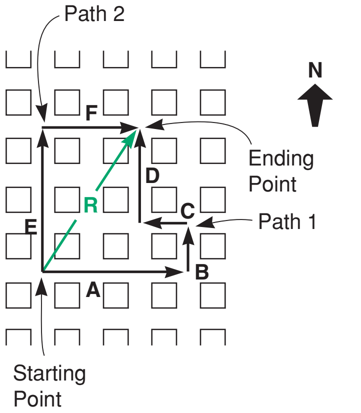
24.



Find the x- and y-components of each vector given in standard position.

25. **A** = 38.9 m at 10.5° 26. **B** = 478 ft at 195.0° 27. **C** = 9.60 km at 310.0°
28. **D** = 5430 mi at 153.7° 29. **E** = 29.5 m at 101.5° 30. **F** = 154 mi at 273.2°

Figure 3.22



The resultant vector **R** is the graphic sum of the component sets of vectors **A**, **B**, **C**, and **D**, and **E** and **F**. That is, **A** + **B** + **C** + **D** = **R** and **E** + **F** = **R**.

3.3 Addition of Vectors

Any given displacement can be the result of many different combinations of displacements. In Fig. 3.22, the displacement represented by the arrow labeled **R** for resultant is the result of either of the two paths shown. The resultant vector, **R**, is the sum of the vectors **A**, **B**, **C**, and **D**. It is also the sum of vectors **E** and **F**. That is,

$$\mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} = \mathbf{R} \quad \text{and} \quad \mathbf{E} + \mathbf{F} = \mathbf{R}$$

To solve a vector addition problem graphically such as displacement:

1. Choose a suitable scale and calculate the length of each vector.
2. Draw the north–south reference line. Graph paper should be used.
3. Using a ruler and protractor, draw the first vector and then draw the other vectors so that the initial end of each vector is placed at the terminal end of the previous vector.
4. Draw the resultant vector from the initial end of the first vector to the terminal end of the last vector.
5. Measure the length of the resultant and use the scale to find the magnitude of the vector. Use a protractor to measure the angle of the resultant.

EXAMPLE 1

Find the resultant displacement of an airplane that flies $2\bar{0}$ mi due east, then $3\bar{0}$ mi due north, and then $1\bar{0}$ mi at 60° west of south.

We choose a scale of 1.0 cm = 5.0 mi so that the vectors are large enough to be accurate and small enough to fit on the paper. (Here each block represents 0.5 cm.) The length of the first vector is

$$|\mathbf{A}| = 2\bar{0}\ \cancel{\text{mi}} \times \frac{1.0\ \text{cm}}{5.0\ \cancel{\text{mi}}} = 4.0\ \text{cm}$$

The length of the second vector is

$$|\mathbf{B}| = 3\bar{0}\ \cancel{\text{mi}} \times \frac{1.0\ \text{cm}}{5.0\ \cancel{\text{mi}}} = 6.0\ \text{cm}$$

The length of the third vector is

$$|\mathbf{C}| = 1\bar{0}\ \cancel{\text{mi}} \times \frac{1.0\ \text{cm}}{5.0\ \cancel{\text{mi}}} = 2.0\ \text{cm}$$

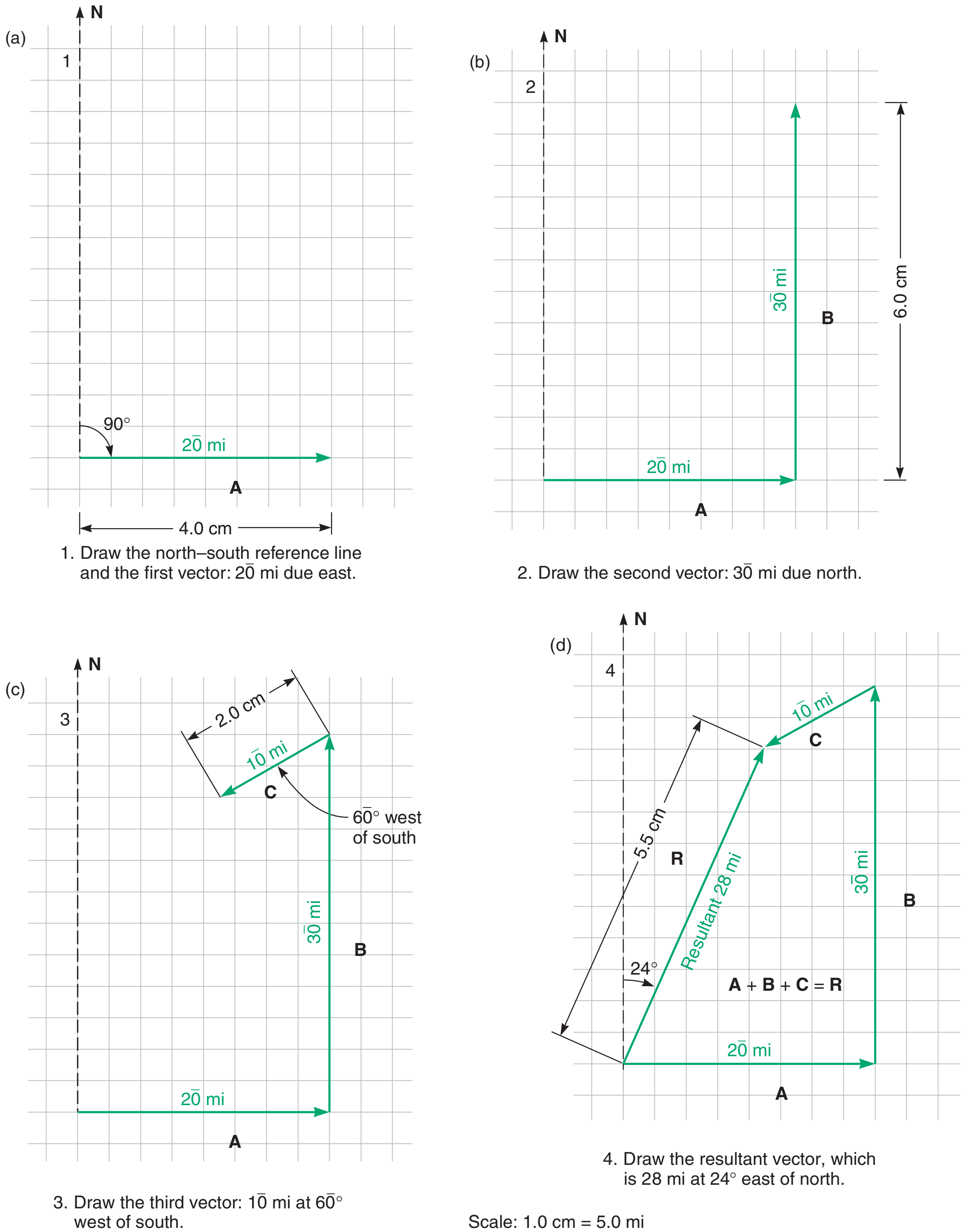
Draw the north–south reference line, and draw the first vector as shown in Fig. 3.23(a). The second and third vectors are then drawn as shown in Fig. 3.23(b) and 3.23(c).

Using a ruler, we find that the length of the resultant vector measures 5.5 cm [Fig. 3.23(d)]. Since 1.0 cm = 5.0 mi, this represents a displacement with magnitude

$$|\mathbf{R}| = 5.5\ \cancel{\text{cm}} \times \frac{5.0\ \text{mi}}{1.0\ \cancel{\text{cm}}} = 28\ \text{mi}$$

The angle between vector **R** and north measures 24°, so the resultant is 28 mi at 24° east of north.

Figure 3.23



Scale: 1.0 cm = 5.0 mi

EXAMPLE 2

Find the resultant of the displacements $15\bar{0}$ km due west, then $20\bar{0}$ km due east, and then 125 km due south.

Choose a scale of 1.0 cm = $5\bar{0}$ km. The length of the first vector is

$$|\mathbf{A}| = 15\bar{0}\ \cancel{\text{km}} \times \frac{1.0\text{ cm}}{5\bar{0}\ \cancel{\text{km}}} = 3.0\text{ cm}$$

The length of the second vector is

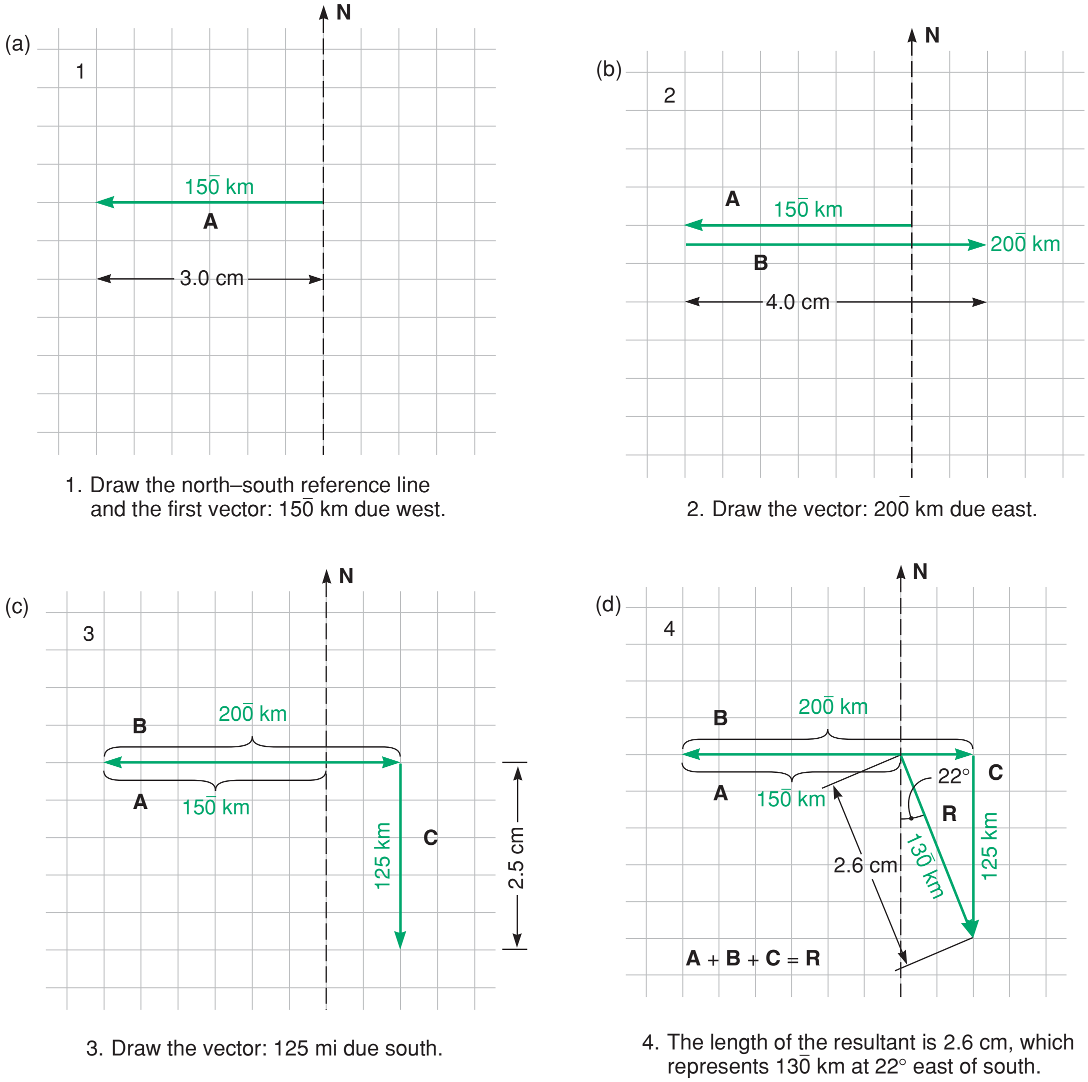
$$|\mathbf{B}| = 20\bar{0}\ \cancel{\text{km}} \times \frac{1.0\text{ cm}}{5\bar{0}\ \cancel{\text{km}}} = 4.0\text{ cm}$$

The length of the third vector is

$$|\mathbf{C}| = 125\ \cancel{\text{km}} \times \frac{1.0\text{ cm}}{5\bar{0}\ \cancel{\text{km}}} = 2.5\text{ cm}$$

Draw the north–south reference line, and draw the first vector as shown in Fig. 3.24(a). Then, draw the second and third vectors as shown in Fig. 3.24(b) and 3.24(c).

Figure 3.24



Scale: 1.0 cm = $5\bar{0}$ km

The length of the resultant vector measures 2.6 cm in Fig. 3.24(d). Since 1.0 cm = $5\bar{0}$ km,

$$|\mathbf{R}| = 2.6\ \cancel{\text{cm}} \times \frac{5\bar{0}\ \text{km}}{1.0\ \cancel{\text{cm}}} = 130\ \text{km}$$

The angle between vector **R** and south measures 22°, so the resultant vector is 130 km at 22° east of south.

Expressing the x- and y-components as signed numbers, we find the resultant vector of several vectors as follows:

1. Find the x-component of each vector and then find the sum of these x-components. This sum is the x-component of the resultant vector.
2. Find the y-component of each vector and then find the sum of these y-components. This sum is the y-component of the resultant vector.

EXAMPLE 3

Given vectors **A** and **B** in Fig. 3.25, graph and find the x- and y-components of the resultant vector **R**.

Graph resultant vector **R** by connecting the initial point of vector **A** to the end point of vector **B** [Fig. 3.26(a)]. The resultant vector **R** is shown in Fig. 3.26(b).

Find the x-component of **R** by finding and adding the x-components of **A** and **B**.

$$\begin{aligned} \mathbf{A}_x &= +3 \\ \mathbf{B}_x &= \underline{+2} \\ \mathbf{R}_x &= +5 \end{aligned}$$

Figure 3.25

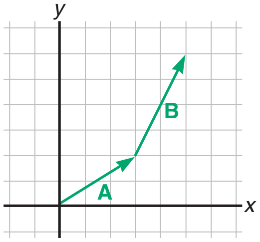
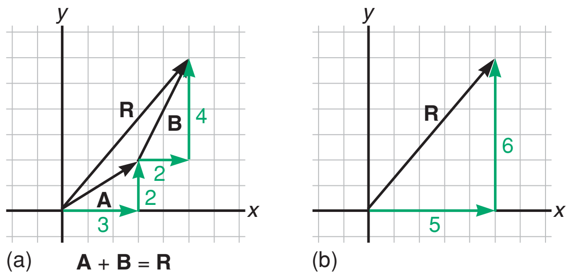


Figure 3.26



Find the y-component of **R** by finding and adding the y-components of **A** and **B**.

$$\begin{aligned} \mathbf{A}_y &= +2 \\ \mathbf{B}_y &= \underline{+4} \\ \mathbf{R}_y &= +6 \end{aligned}$$

EXAMPLE 4

Given vectors **A**, **B**, and **C** in Fig. 3.27, graph and find the x- and y-components of the resultant vector **R**.

Graph resultant vector **R** by connecting the initial point of vector **A** to the end point of vector **C** [Fig. 3.28(a)]. The resultant vector **R** is shown in Fig. 3.28(b).

Figure 3.27

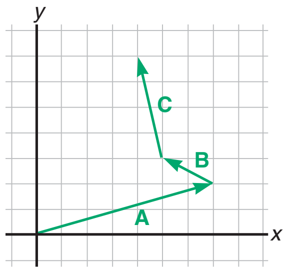
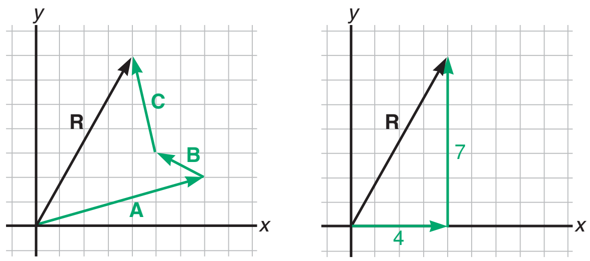


Figure 3.28



(a) A + B + C = R (b)

Find the x-component of **R** by finding and adding the x-components of **A**, **B**, and **C** as shown below. Find the y-component of **R** by finding and adding the y-components of **A**, **B**, and **C**.

Vector	x-component	y-component
A	+7	+2
B	−2	+1
C	−1	+4
R	+4	+7

EXAMPLE 5

Given vectors **A**, **B**, **C**, and **D** in Fig. 3.29, graph and find the x- and y-components of the resultant vector **R**.

Graph resultant vector **R** by connecting the initial point of vector **A** to the end point of vector **D** [Fig. 3.30(a)]. The resultant vector **R** is shown in Fig. 3.30(b).

Figure 3.29

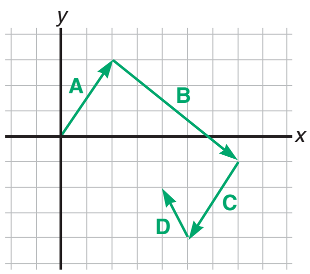
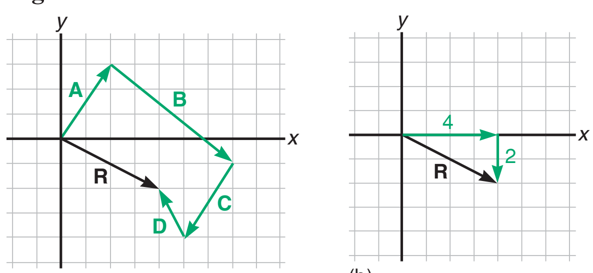


Figure 3.30



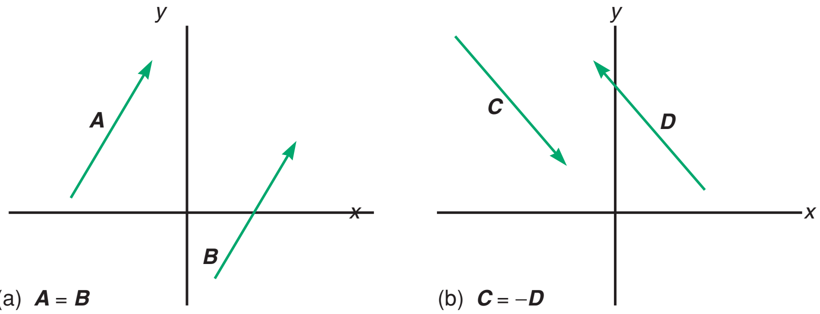
(a) A + B + C + D = R (b)

Find the x-component of **R** by finding and adding the x-components of **A**, **B**, **C**, and **D** as shown below. Find the y-component of **R** by finding and adding the y-components of **A**, **B**, **C**, and **D**.

Vector	x-component	y-component
A	+2	+3
B	+5	−4
C	−2	−3
D	−1	+2
R	+4	−2

Two vectors are equal when they have the same magnitude and the same direction [Fig. 3.31(a)]. *Two vectors are opposites or negatives* of each other when they have the same magnitude but opposite directions [Fig. 3.31(b)].

Figure 3.31



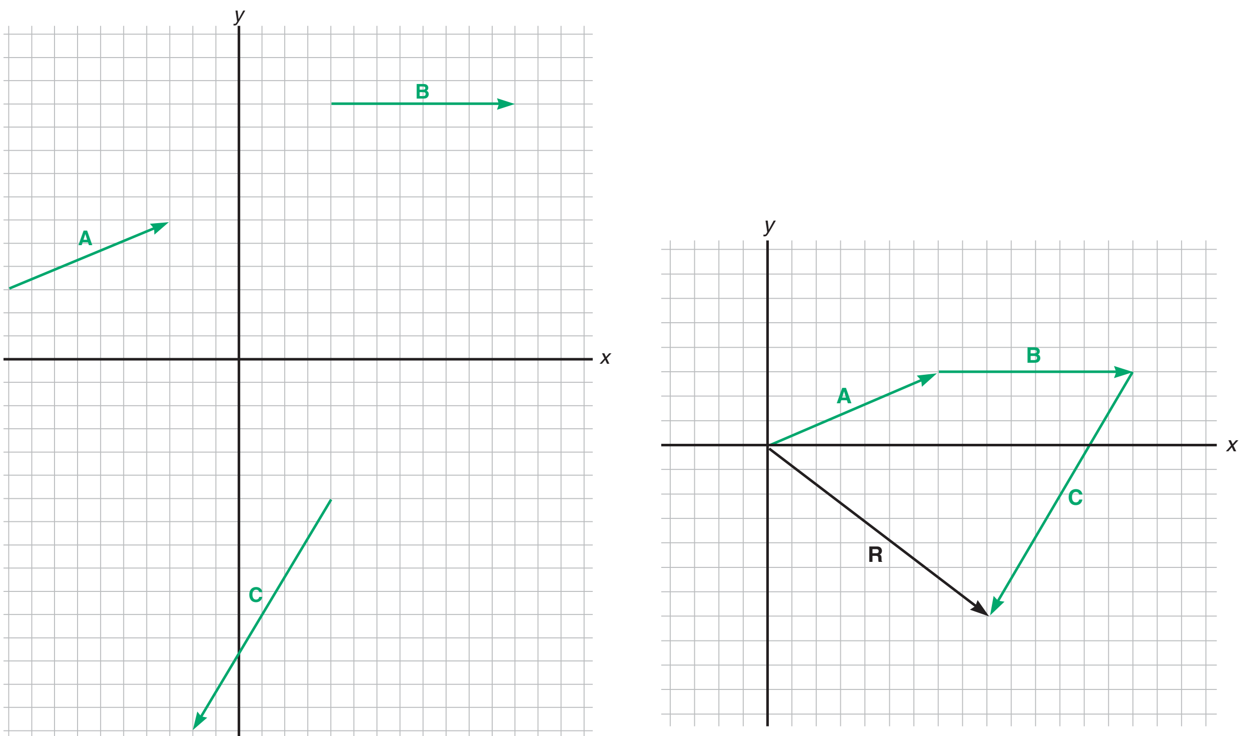
(a) ***A*** = ***B*** (b) ***C*** = −***D***

To add two or more vectors in any position graphically, construct the first vector with its initial point at the origin and parallel to its given position. Then, construct the second vector with its initial point on the end point of the first vector and parallel to its given position. Then, construct the third vector with its initial point on the end point of the second vector and parallel to its given position. Continue this process until all vectors have been so constructed. The resultant vector is the vector joining the initial point of the first vector (origin) to the end point of the last vector. (The order of adding or constructing the given vectors does not matter.)

EXAMPLE 6

Given vectors **A**, **B**, and **C** in Fig. 3.32(a), graph and find the *x*- and *y*-components of the resultant vector **R**.

Figure 3.32



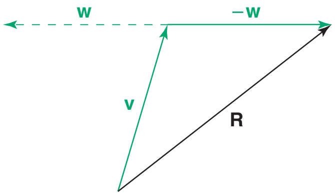
A + B + C = R

(a) (b)

Construct vector **A** with its initial point at the origin and parallel to its given position as in Fig. 3.32(b). Next, construct vector **B** with its initial point on the end point of vector **A** and parallel to its given position. Then, construct vector **C** with its initial point on the end point of vector **B** and parallel to its given position. The resultant vector **R** is the vector with its initial point at the origin and its end point at the end point of vector **C**.

From the graph in Fig. 3.32(b), we read the x-component of **R** as $+9$ by counting the number of squares *to the right* between the y-axis and the end point of vector **R**. We read the y-component of **R** as -7 by counting the number of squares *below* and between the x-axis and the end point of vector **R**.

Figure 3.33
$\mathbf{R} = \mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w})$



One vector may be subtracted from a second vector by adding its negative to the first. That is, $\mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w})$. Construct **v** as usual. Then construct $-\mathbf{w}$ and find the resultant **R** as shown in Fig. 3.33.

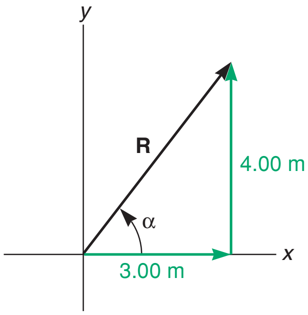
EXAMPLE 7

Finding a Vector from Its Components

Find vector **R** in standard position with $\mathbf{R}_x = +3.00$ m and $\mathbf{R}_y = +4.00$ m.

First, graph the x- and y-components (Fig. 3.34) and complete the right triangle. The hypotenuse is the resultant vector. Find angle α as follows:

Figure 3.34



$$\tan \alpha = \frac{\text{side opposite } \alpha}{\text{side adjacent to } \alpha}$$

$$\tan \alpha = \frac{4.00 \text{ m}}{3.00 \text{ m}} = 1.333$$

$$\alpha = 53.1° \qquad \text{(see Appendix B, Section B.3)}$$

Find the magnitude of **R** using the Pythagorean theorem:

$$|\mathbf{R}| = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$$

$$|\mathbf{R}| = \sqrt{(3.00 \text{ m})^2 + (4.00 \text{ m})^2} = 5.00 \text{ m}$$

That is, $\mathbf{R} = 5.00$ m at 53.1°.

In general:

To find resultant vector **R** in standard position when its x- and y-components are given:

1. Complete the right triangle with the legs being the x- and y-components of the vector.
2. Find the acute angle α of the right triangle whose vertex is at the origin by using $\tan \alpha$.
3. Find angle θ in standard position as follows:

$$\theta = \alpha \qquad (\theta \text{ in first quadrant})$$
$$\theta = 180° - \alpha \qquad (\theta \text{ in second quadrant})$$
$$\theta = 180° + \alpha \qquad (\theta \text{ in third quadrant})$$
$$\theta = 360° - \alpha \qquad (\theta \text{ in fourth quadrant})$$

   The Greek letter θ (theta) is often used to represent the measure of an angle.

4. Find the magnitude of the vector using the Pythagorean theorem:

$$\mathbf{R} = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$$

EXAMPLE 8

Find vector **R** in standard position whose x-component is $+7.00$ mi and y-component is -5.00 mi.

First, graph the x- and y-components (Fig. 3.35) and complete the right triangle. The hypotenuse is the resultant vector. Find angle α as follows:

$$\tan \alpha = \frac{\text{side opposite } \alpha}{\text{side adjacent to } \alpha}$$

$$\tan \alpha = \frac{5.00 \text{ mi}}{7.00 \text{ mi}} = 0.7143$$

$$\alpha = 35.5°$$

Then

$$\begin{aligned} \theta &= 360° - \alpha \qquad \textbf{R} \text{ is in the fourth quadrant.} \\ &= 360° - 35.5° \\ &= 324.5° \end{aligned}$$

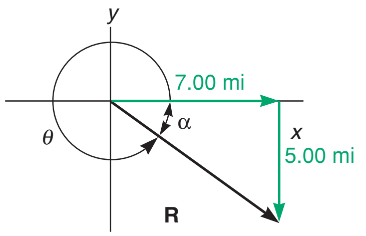
Find the magnitude of **R** using the Pythagorean theorem:

$$|\mathbf{R}| = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$$

$$\begin{aligned} |\mathbf{R}| &= \sqrt{|7.00 \text{ mi}|^2 + |-5.00 \text{ mi}|^2} \\ &= 8.60 \text{ mi} \end{aligned}$$

That is, **R** = 8.60 mi at 324.5°.

Figure 3.35



EXAMPLE 9

Find vector **R** in standard position with $\mathbf{R}_x = -115$ km and $\mathbf{R}_y = +175$ km.

First, graph the x- and y-components (Fig. 3.36) and complete the right triangle. The hypotenuse is the resultant vector. Find angle α as follows:

$$\tan \alpha = \frac{\text{side opposite } \alpha}{\text{side adjacent to } \alpha}$$

$$\tan \alpha = \frac{175 \text{ km}}{115 \text{ km}} = 1.522$$

$$\alpha = 56.7°$$

Then

$$\begin{aligned} \theta &= 180° - \alpha \qquad \textbf{R} \text{ is in the second quadrant.} \\ &= 180° - 56.7° \\ &= 123.3° \end{aligned}$$

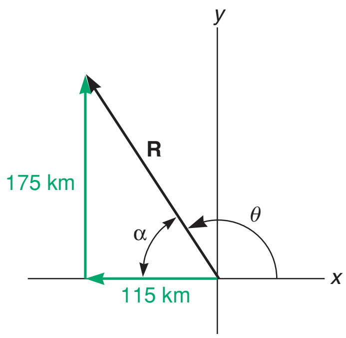
Find the magnitude of **R** using the Pythagorean theorem:

$$|\mathbf{R}| = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$$

$$\begin{aligned} |\mathbf{R}| &= \sqrt{|-115 \text{ km}|^2 + |175 \text{ km}|^2} \\ &= 209 \text{ km} \end{aligned}$$

That is, **R** = 209 km at 123.3°.

Figure 3.36



To find the resultant vector **R** of any set of vectors, such as **R** = **A** + **B** + **C**, using right-triangle trigonometry:

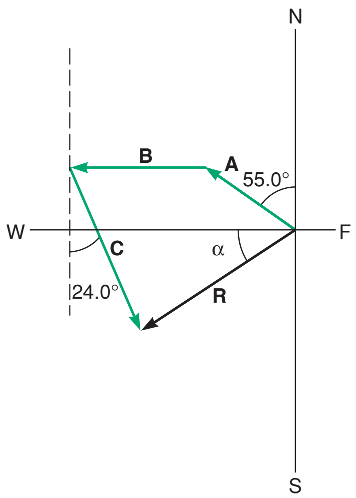
1. Find the x-component of each vector and add: $\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x + \mathbf{C}_x$.
2. Find the y-component of each vector and add: $\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y + \mathbf{C}_y$.
3. Find the magnitude of the resultant vector **R** using the Pythagorean theorem $|\mathbf{R}| = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$.
4. Find the direction of the resultant vector **R** using right-triangle trigonometry by (a) first finding the acute α between the resultant vector and the x-axis and then finding angle θ in standard position or (b) expressing the direction of the resultant vector using some other reference.

EXAMPLE 10

A ship travels 105 km from port on a course of 55.0° west of north to an island. Then it travels 124 km due west to a second island. Then it travels 177 km on a course of 24.0° east of south to a third island. Find the displacement from the starting point to the ending point.

First, draw a vector diagram as in Fig. 3.37. Then, find the x- and y-components of each of the three vectors as follows:

Figure 3.37



$$|\mathbf{A}_x| = |\mathbf{A}| \cos 35.0°$$ The acute angle between vector **A** and the x-axis is $90° - 55.0° = 35.0°$.

$$= (105 \text{ km})(\cos 35.0°)$$
$$= -86.0 \text{ km}$$ The x-component is in the negative x-direction.

$$|\mathbf{A}_y| = |\mathbf{A}| \sin 35.0°$$
$$= (105 \text{ km})(\sin 35.0°)$$
$$= 60.2 \text{ km}$$ The y-component is in the positive y-direction.

$$|\mathbf{B}_x| = |\mathbf{B}| = -124 \text{ km}$$ This x-component is in the negative x-direction.

$$|\mathbf{B}_y| = |\mathbf{B}| = 0 \text{ km}$$ This y-component of due west is 0.

$$|\mathbf{C}_x| = |\mathbf{C}| \cos 66.0°$$ The acute angle between vector **C** in standard position and the x-axis is $90° - 24.0° = 66.0°$.

$$= (177 \text{ km})(\cos 66.0°)$$
$$= 72.0 \text{ km}$$ The x-component is in the positive x-direction.

$$|\mathbf{C}_y| = |\mathbf{C}| \sin 66.0°$$
$$= (177 \text{ km})(\sin 66.0°)$$
$$= -162 \text{ km}$$ The y-component is in the negative y-direction.

Thus

$$\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x + \mathbf{C}_x = -86.0 \text{ km} + (-124 \text{ km}) + 72.0 \text{ km} = -138 \text{ km}$$
$$\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y + \mathbf{C}_y = 60.2 \text{ km} + 0 + (-162 \text{ km}) = -102 \text{ km}$$
$$|\mathbf{R}| = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$$
$$= \sqrt{|-138 \text{ km}|^2 + |-102 \text{ km}|^2}$$
$$= 172 \text{ km}$$
$$\tan \alpha = \frac{102 \text{ km}}{138 \text{ km}} = 0.7391$$
$$\alpha = 36.5°$$

So, the displacement is 172 km at 36.5° south of west. That is, the ship stops at a port that is 172 km at 36.5° south of west from its starting point.

PROBLEMS 3.3

Use graph paper to find the resultant of each displacement pair.

1. 35 km due east, then $5\overline{0}$ km due north
2. $6\overline{0}$ km due west, then $9\overline{0}$ km due south
3. $5\overline{0}0$ mi at 75° east of north, then $15\overline{0}0$ mi at $2\overline{0}$° west of south
4. $2\overline{0}$ mi at 3° north of east, then 17 mi at 9° west of south
5. 67 km at 55° north of west, then 46 km at 25° south of east
6. 4.0 km at 25° west of south, then 2.0 km at 15° north of east

Use graph paper to find the resultant of each set of displacements.

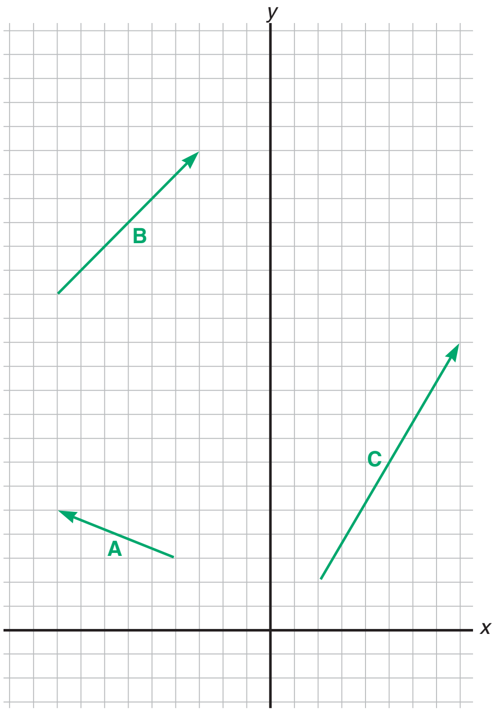
7. $6\overline{0}$ km due south, then $9\overline{0}$ km at 15° north of west, and then 75 km at 45° north of east
8. 110 km at $5\overline{0}$° north of east, then 170 km at $3\overline{0}$° east of south, and then 145 km at $2\overline{0}$° north of east
9. 1700 mi due north, then 2400 mi at $1\overline{0}$° north of east, and then $2\overline{0}00$ mi at $2\overline{0}$° south of west
10. $9\overline{0}$ mi at $1\overline{0}$° west of north, then 75 mi at $3\overline{0}$° west of south, and then 55 mi at $2\overline{0}$° east of south
11. 75 km at 25° north of east, then 75 km at 65° south of west, and then 75 km due south
12. 17 km due north, then $1\overline{0}$ km at 7° south of east, and then 15 km at $1\overline{0}$° west of south
13. 12 mi at 58° north of east, then 16 mi at 78° north of east, then $1\overline{0}$ mi at 45° north of east, and then 14 mi at $1\overline{0}$° north of east
14. $1\overline{0}$ km at 15° west of south, then 27 km at 35° north of east, then 31 km at 5° north of east, and then 22 km at $2\overline{0}$° west of north

Find the *x*- and *y*-components of each resultant vector **R** and graph the resultant vector **R**.

	Vector	*x*-component	*y*-component		Vector	*x*-component	*y*-component
15.	**A**	+2	+3	16.	**A**	+9	−5
	B	+7	+2		**B**	−4	−6
	R				**R**		
17.	**A**	−2	+13	18.	**A**	+10	−5
	B	−11	+1		**B**	−13	−9
	C	+3	−4		**C**	+4	+3
	R				**R**		
19.	**A**	+17	+7	20.	**A**	+1	+7
	B	−14	+11		**B**	+9	−4
	C	+7	+9		**C**	−4	+13
	D	−6	−15		**D**	−11	−4
	R				**R**		
21.	**A**	+1.5	−1.5	22.	**A**	+1	−1
	B	−3	−2		**B**	−4	−2
	C	+7.5	−3		**C**	+2	+4
	D	+2	+2.5		**D**	+5	−3
	R				**E**	+3	+5
					R		
23.	**A**	+1.5	+2.5	24.	**A**	−7	+15
	B	−2	−3		**B**	+13.5	−17.5
	C	+3.5	−7.5		**C**	−7.5	−20
	D	−4	+6		**D**	+6	+13.5
	E	−5.5	+2		**E**	+2.5	+2.5
	R				**F**	−11	+11.5
					R		

For each set of vectors, graph and find the *x*- and *y*-components of the resultant vector **R**.

25.



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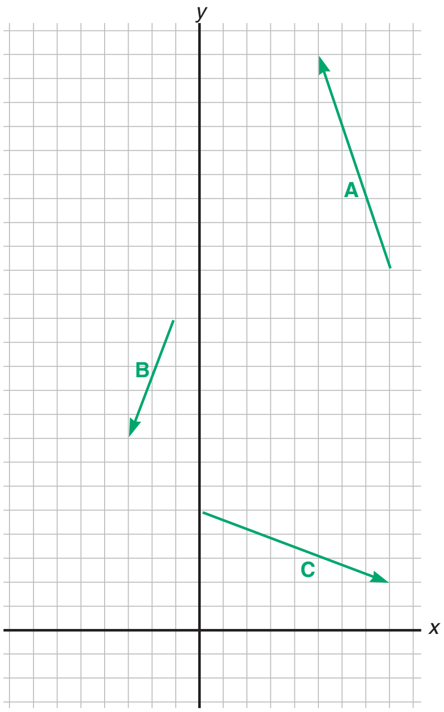

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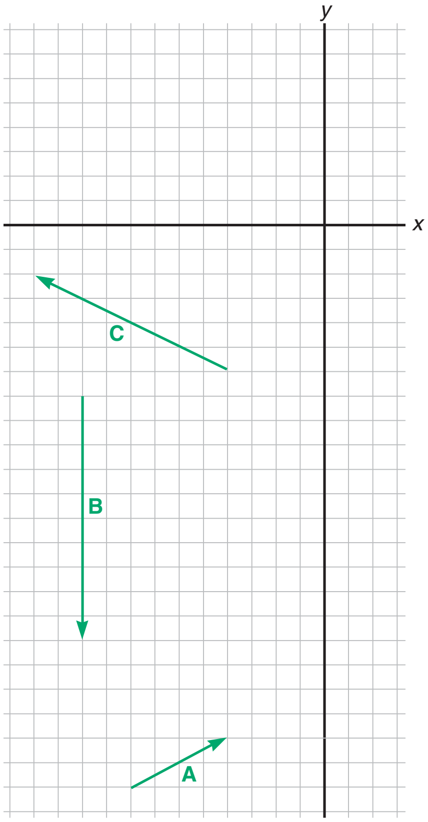

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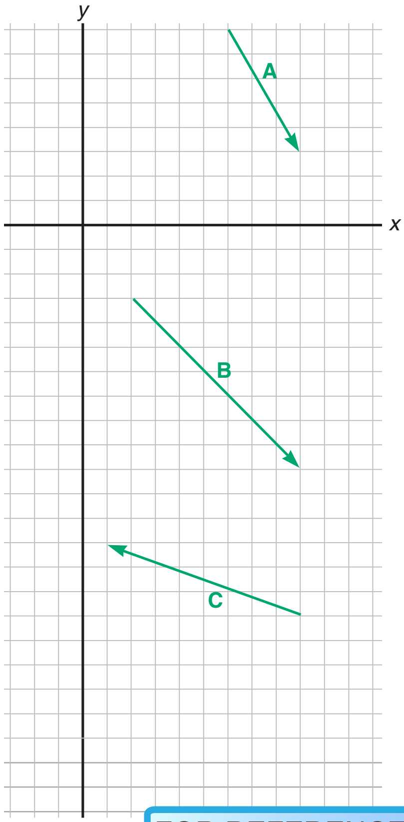

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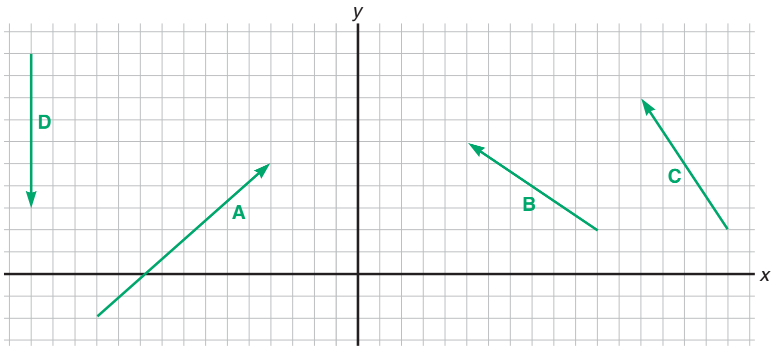

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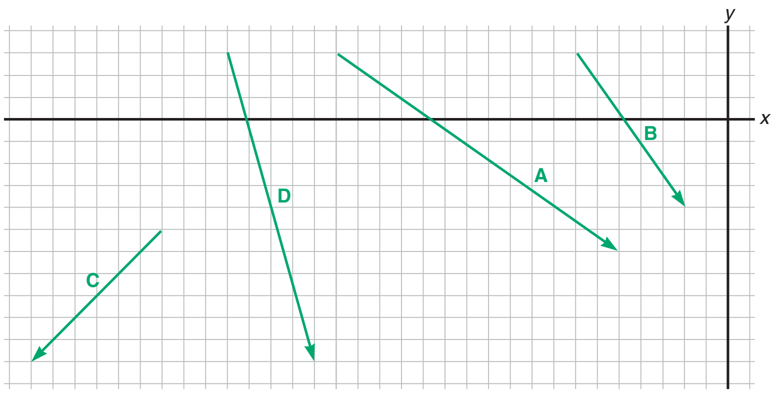
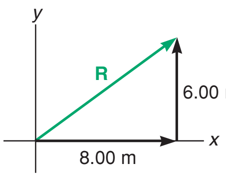


In Problems 31 through 42, find each resultant vector **R**. Give **R** in standard position.

31.



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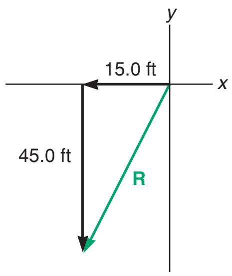

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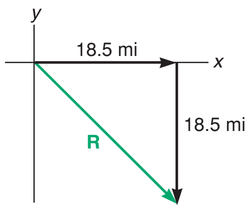

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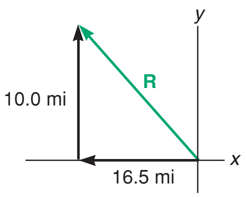

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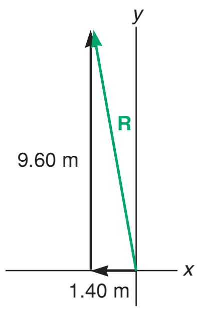

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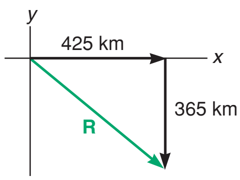


	x-component	*y-component*		*x-component*	*y-component*
37.	+19.5 m	−49.6 m	38.	−158 km	+236 km
39.	+14.7 mi	+16.8 mi	40.	−3240 ft	−1890 ft
41.	−9.65 m	+4.36 m	42.	+375 km	−408 km

43. A road grader must go around a pond by traveling $10\bar{0}$ m south and then $15\bar{0}$ m east. If the road grader could go directly from the beginning point to the end point, how far would it travel?

44. An earthmover must go north $35\bar{0}$ m and then west 275 m to avoid a pipeline hazard. What distance would it travel if it could go directly to the endpoint?

45. An airplane takes off and flies 225 km on a course of 25.0° north of west and then changes direction and flies 135 km due north where it lands. Find the displacement from its starting point to its landing point.

46. A ship travels 50.0 mi on a course of 15.0° south of east and then travels 85.5 mi on a course of 60.0° west of south. Find the displacement from its starting point to its ending point.
47. A ship travels 135 km from port on a course of 25.0° south of east to an island. It then travels 122 km on a course of 35.5° west of south to a second island. Then it travels 135 km on a course of 10.4° north of west to a third island. Find the displacement from its starting point to its ending point.
48. A ship travels 145 km from port on a course of 65.0° north of east to an island. It then travels 112 km on a course of 30.5° west of north to a second island. Then it travels 182 km on a course of 10.4° west of south to a third island. Then it travels 42.5 km due south to a fourth island. Find the displacement from its starting point to its ending point.

PHYSICS CONNECTIONS

Global Positioning Satellites

Navigators continually struggle to find better tools to help them determine their location. The first explorers used the sun and stars to help them steer a straight course, but this method of navigation only worked under clear skies. Magnetic compasses were developed, yet could only be used to determine longitude, not latitude. Finally, the mechanical clock, in conjunction with the compass, provided navigators with the most accurate method of determining location. Today, most navigators use a hand-held device that functions in concert with a series of 24 orbiting satellites. This network, the Global Positioning System (GPS), can determine your position and altitude anywhere on earth.

The GPS pinpoints your location by sending out radio signals to locate any 4 of the 24 orbiting GPS satellites. Once the satellites are found, the GPS measures the length of time it takes for a radio signal to reach the hand-held receiver. When the time is determined for each of 4 satellites, the distance is calculated, and the longitude, latitude, and altitude are displayed on the screen [Fig. 3.38(a)].

GPS was first developed solely for military use. Eventually, the GPS system was made available for civilian businesses. Shipping, airline, farming, surveying, and geological companies made use of the technology. Today, GPS receivers are affordable and are used by the general public [Fig. 3.38(b)]. More sophisticated receivers not only locate a position, but can also guide the navigator to a predetermined location. Several automobile manufacturers have included GPS receivers as an option in their cars. Such receivers come complete with voice commands such as, "Turn left at the next traffic light," as part of their option packages.

Figure 3.38 (a) The screen on the GPS receiver shows the position and strength of the signal between the receiver and the various satellites. At the time this photograph was taken, the receiver picked up 7 of the 12 overhead satellites, bringing the precision to within 20 ft of the actual location. (b) Global Positioning Systems have allowed for an enormous step forward in navigation. The GPS receiver shown has monitored and recorded precisely where the person has traveled and is now helping the user find his way back to camp.

(a)

(b)

Glossary

Component Vector When two or more vectors are added, each of the vectors is called a component of the resultant, or sum, vector. (p. 72)

Displacement The net change in position of an object, or the direct distance and direction it moves; a vector. (p. 68)

Number Plane A plane determined by the horizontal line called the x-axis and a vertical line called the y-axis intersecting at a right angle at a point called the origin. These two lines divide the number plane into four quadrants. The x-axis contains positive numbers to the right of the origin and negative numbers to the left of the origin. The y-axis contains positive numbers above the origin and negative numbers below the origin. (p. 71)

Resultant Vector The sum of two or more vectors. (p. 72)

Scalar A physical quantity that can be completely described by a number (called its magnitude) and a unit. (p. 68)

Standard Position A vector is in standard position when its initial point is at the origin of the number plane. The vector is expressed in terms of its length and its angle θ, where θ is measured counterclockwise from the positive x-axis to the vector. (p. 73)

Vector A physical quantity that requires both magnitude (size) and direction to be completely described. (p. 68)

***x*-component** The horizontal component of a vector that lies along the x-axis. (p. 72)

***y*-component** The vertical component of a vector that lies along the y-axis. (p. 72)

Formulas

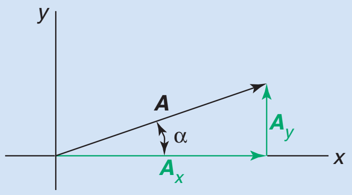
3.2 To find the x- and y-components of a vector $\mathbf{v}$ given in standard position (Fig. 3.39):

1. Complete the right triangle with the legs being the x- and y-components of the vector.
2. Find the lengths of the legs of the right triangle as follows:

$$|\mathbf{A}_x| = |\mathbf{A}|(\cos\alpha)$$
$$|\mathbf{A}_y| = |\mathbf{A}|(\sin\alpha)$$

3. Determine the signs of the x- and y-components.

Figure 3.39



3.3 To find resultant vector $\mathbf{R}$ in standard position when its x- and y-components are given:

1. Complete the right triangle with the legs being the x- and y-components of the vector.
2. Find the acute angle α of the right triangle whose vertex is at the origin by using $\tan\alpha$.
3. Find angle θ in standard position as follows:

$$\begin{aligned}
\theta &= \alpha && (\theta \text{ in first quadrant})\\
\theta &= 180° - \alpha && (\theta \text{ in second quadrant})\\
\theta &= 180° + \alpha && (\theta \text{ in third quadrant})\\
\theta &= 360° - \alpha && (\theta \text{ in fourth quadrant})
\end{aligned}$$

4. Find the magnitude of the vector using the Pythagorean theorem:

$$\mathbf{R} = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$$

To find the resultant vector **R** of any set of vectors, such as $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C}$, using right-triangle trigonometry:

1. Find the x-component of each vector and add: $\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x + \mathbf{C}_x$.
2. Find the y-component of each vector and add: $\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y + \mathbf{C}_y$.
3. Find the magnitude of the resultant vector **R** using the Pythagorean theorem $|\mathbf{R}| = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$.
4. Find the direction of the resultant vector **R** using right-triangle trigonometry by (a) first finding the acute angle α between the resultant vector and the x-axis and then finding angle θ in standard position or (b) expressing the direction of the resultant vector using some other reference.

Review Questions

1. Displacement
   (a) can be interchanged with direction.
   (b) is a measurement of volume.
   (c) can be described only with a number.
   (d) is the net distance an object travels, showing direction and distance.
2. When adding vectors, the order in which they are added
   (a) is not important. (b) is important.
   (c) is important only in certain cases.
3. A vector is in standard position when its initial point is
   (a) at the origin. (b) along the x-axis.
   (c) along the y-axis.
4. Discuss number plane, origin, and axis in your own words.
5. Can every vector be described in terms of its components?
6. Can a vector have more than one set of component vectors?
7. Describe how to add two or more vectors graphically.
8. Describe how to find a resultant vector if given its x- and y-components.
9. Is a vector limited to a single position in the number plane?
10. Is the angle of a vector in standard position measured clockwise or counterclockwise?
11. What are the limits on the angle measure of a vector in standard position in the third quadrant?
12. Describe how to find the x- and y-components of a vector given in standard position.
13. Describe how to find a vector in standard position when the x- and y-components are given.

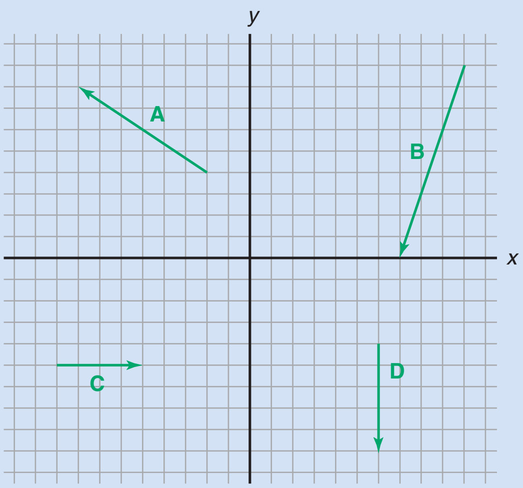
Review Problems

1. Find the x- and y-components of vector **R**, which has a length of 13.0 cm at 30.0°.
2. Find the x- and y-components of vector **R**, which has a length of 10.0 cm at 60.0°.
3. Find the x- and y-components of vector **R**, which has a length of 20.0 cm at 30.0°.
4. Vector **R** has length 9.00 cm at 240.0°. Find its x- and y-components.
5. Vector **R** has length 9.00 cm at 40.0°. Find its x- and y-components.
6. Vector **R** has length 18.0 cm at 305.0°. Find its x- and y-components.
7. A hiker is plotting his course on a map with a scale of 1.00 cm = 3.00 km. If the hiker walks 2.50 cm north, then turns south and walks 1.50 cm, what is the actual displacement of the hiker in km?

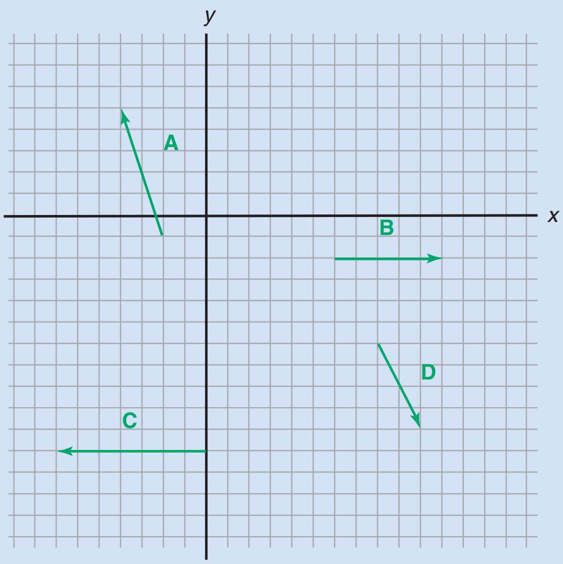
8. A hiker is plotting his course on a map with a scale of 1.00 cm = 3.00 km. If the hiker walks 1.50 cm north, then turns south and walks 2.50 cm, what is the actual displacement of the hiker in km?
9. A co-pilot is charting her course on a map with a scale of 1.00 cm = 20.0 km. If the plane is charted to head 13.0 cm west, 9.00 cm north, and 2.00 cm east, what is the actual displacement of the plane in km?
10. A co-pilot is charting her course on a map with a scale of 1.00 cm = 20.0 km. If the plane is charted to head 25.0° north of east for 16.0 cm, north for 6.00 cm, and west for 5.00 cm, what is the actual displacement of the plane in km?
11. Vector **R** has x-component = +14.0 and y-component = +3.00. Find its length.
12. Vector **R** has x-component = −5.00 and y-component = +10.0. Find its length.
13. Vector **R** has x-component = +8.00 and y-component = −2.00. Find its length.
14. Vector **R** has x-component = −3.00 and y-component = −4.00. Find its length.
15. Vectors **A**, **B**, and **C** are given. Vector **A** has x-component = +3.00 and y-component = +4.00. Vector **B** has x-component = +5.00 and y-component = −7.00. Vector **C** has x-component = −2.00 and y-component = +1.00. Find the resultant vector **R**.
16. Vectors **A**, **B**, and **C** are given. Vector **A** has x-component = +5.00 and y-component = +7.00. Vector **B** has x-component = +9.00 and y-component = −3.00. Vector **C** has x-component = −5.00 and y-component = +5.00. Find the x- and y-components of the resultant vector **R**.
17. Vectors **A**, **B**, and **C** are given. Vector **A** has x-component = −3.00 and y-component = −4.00. Vector **B** has x-component = −5.00 and y-component = +7.00. Vector **C** has x-component = +2.00 and y-component = −1.00. Find the x- and y-components of the resultant vector **R**.
18. Vectors **A**, **B**, and **C** are given. Vector **A** has x-component = −5.00 and y-component = −7.00. Vector **B** has x-component = −9.00 and y-component = +3.00. Vector **C** has x-component = +5.00 and y-component = −5.00. Find the x- and y-components of the resultant vector **R**.

Graph and find the x- and y-components of each resultant vector **R**, where **R** = **A** + **B** + **C** + **D**.

19.



20.



21. An airplane takes off and flies 245 km on a course of 45.0° south of west and then changes direction and flies 175 km due south, where it lands. Find the displacement from its starting point to its landing point.
22. A ship travels 155 km from port on a course of 35.0° south of west to an island. It then travels 142 km on a course of 55.5° east of south to a second island. Then it travels 138 km on a course of 9.4° north of east to a third island. Then it travels 185 km due east to a fourth island. Find the displacement from its starting point to its ending point.

APPLIED CONCEPTS

1. The New Clark Bridge is an elegant cable-stayed bridge. Its design requires cables to reach from the road deck up to the tower and back down to the road deck on the other side of the tower, as shown in Fig. 3.40. In order to determine the best method for shipping the cables, the shipping company needs to know the lengths of the shortest and longest cables. Given the measurements in the diagram, determine the indicated total lengths BEC and AED, respectively.

2. Frank just learned that the $80\overline{0}$-m section of Broadway that he uses to get to work will be closed for several days. Given the information from a map of Manhattan (Fig. 3.41), what is the distance of Frank's next shortest route?

Figure 3.40

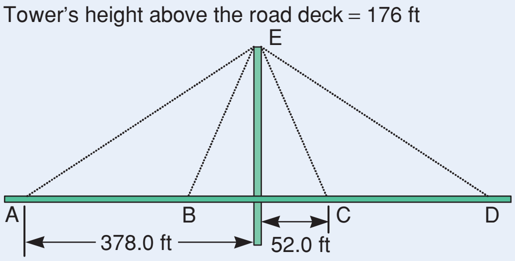
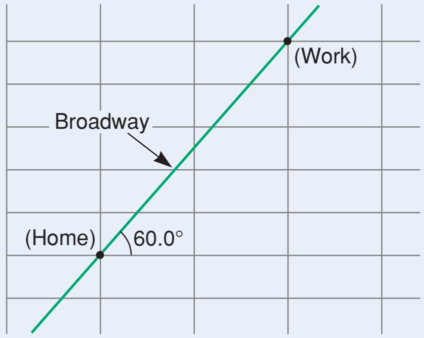
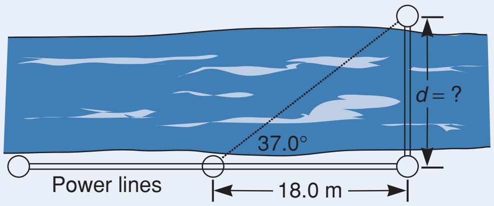


Figure 3.41



3. Power cables need to be suspended by the power company across a river to a new condominium development. Find the distance across the river in Fig. 3.42.

Figure 3.42



4. Bill has set his GPS to track his route. At the conclusion of his hike, the receiver indicates that he walked 3.50 mi north, 1.00 mi northeast, and 1.50 mi south. How far away is Bill from his original position?

5. With the airplane cruising at $30,\overline{0}00$ ft, the navigator indicates to the captain that the plane should continue traveling north for $50\overline{0}$ km and then turn to a heading of 45.0° east of north for $20\overline{0}$ km. What will be the resultant distance traveled?